Omvendt Pythagoras (Euclid I,48)

Sætningen

Givet: en trekant *ABC* med siderne *a*, *b* og *c*. For sidelængderne gælder det, at:

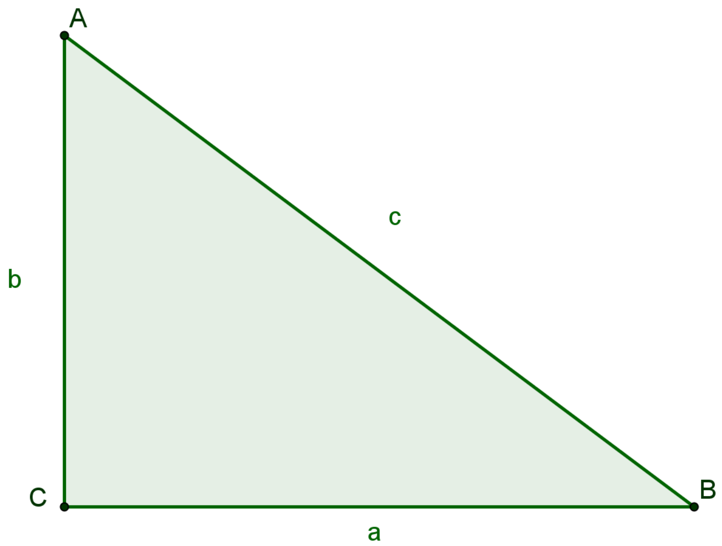
$c^2 = a^2 + b^2$

Så gælder det, at trekanten er retvinklet, med *C* som den rette vinkel.

Bevis

I denne trekant gælder, at

$$c^2 = a^2 + b^2 .$$

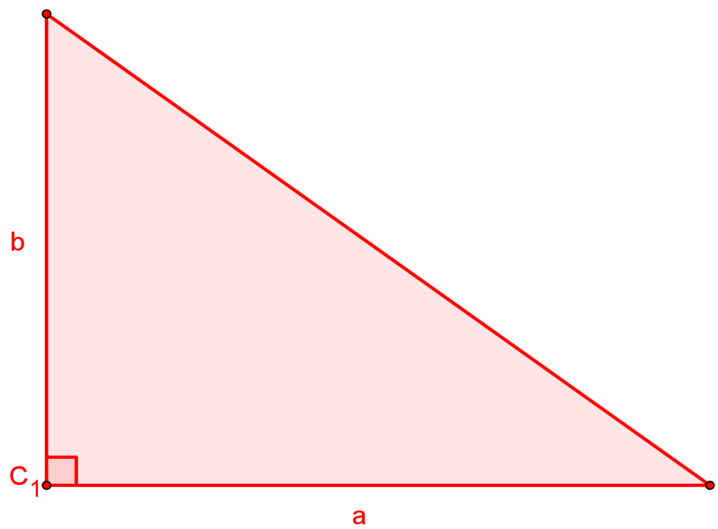


Vi konstruerer nu en ny retvinklet trekant (se herunder), hvor kateterne har de samme længder som i den givne trekant.

Da denne trekant er retvinklet, gælder Pythagoras sætning (Euclid I,47), dvs.

$$c_1^2 = a^2 + b^2$$

Da følger $c_1 = c$, dvs. de to trekanter har alle sider parvis lige store.



Derfor er de kongruente (Euclid I,8) og det vil sige, at vinklerne C og C_1 er lige store. Men så er trekant *ABC* retvinklet.

QED

Appendix: Kongruenssætning SSS

Euclid's Elements, Book I, Proposition 8 - Mozilla Firefox

Filer Rediger Vis Historik Bogmærker Funktioner Hjælp

Euclid's Elements, Book I, Proposition 8

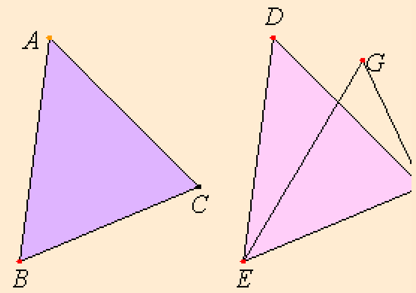
aleph0.clarku.edu/~djoyce/java/elements/bookI/propI8.html

joyce euclid

Euclid's Elements
Book I
Proposition 8

If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.

Let *ABC* and *DEF* be two triangles having the two sides *AB* and *AC* equal to the two sides *DE* and *DF* respectively, namely *AB* equal to *DE* and *AC* equal to *DF*, and let them have the base *BC* equal to the base *EF*.



I say that the angle *BAC* also equals the angle *EDF*.

If the triangle *ABC* is applied to the triangle *DEF*, and if the point *B* is placed on the point *E* and the straight line *BC* on *EF*, then the point *C* also coincides with *F*, because *BC* equals *EF*.

Then, *BC* coinciding with *EF*, therefore *BA* and *AC* also coincide with *ED* and *DF*, for, if the base *BC* coincides with the base *EF*, and the sides *BA* and *AC* do not coincide with *ED* and *DF* but fall beside them as *EG* and *GF*, then given two straight lines constructed on a straight line and meeting in a point, there will have been constructed on the same straight line and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same end with it.

But they cannot be so constructed. I.7

Therefore it is not possible that, if the base *BC* is applied to the base *EF*, the sides *BA* and *AC* do not coincide with *ED* and *DF*. Therefore they coincide, so that the angle *BAC* coincides with the angle *EDF*, and equals it. C.N.4

Therefore *if two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.*

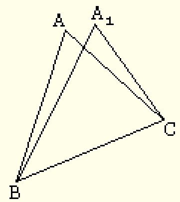
Q.E.D.

initializing Geometry applet

A proof that combines I.8 and, in a sense, I.7 is given in the famous text by J. Hadamard.

Proof 3 (J. Hadamard)

Two triangles are equal if they have three respectively equal sides.



Let there be two triangle ABC and A'B'C', whose sides are respectively equal. Let's place the second triangle so as to make side B'C' coincide with BC and have both triangles lie on the same side of BC. Let BCA_1 be the new position of triangle B'C'A'. I claim that point A_1 coincides with point A. It would have been evident had line B'A' had the direction of BA or line C'A' the direction of CA. Had it not been so, we would have had two isosceles triangles BAA_1 and CAA_1, and the perpendicular bisector of AA_1 would have to pass through points B and C; in other words, the perpendicular bisector of AA_1 would have to coincide with the line BC. But this is impossible, since points A and A_1 lie on the same side of BC, and therefore BC can't pass through the midpoint of AA_1. The points A and A_1 can't therefore be different. The two triangles are bound to be equal.